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Free convection heat transfer from horizontal wires to pseudoplastic fluids

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Abstract—Experimental Nusselt values are reported for free convection heat transfer from small diameter horizontal wires to pseudoplastic fluids.

INTRODUCTION

FREE CONVECTION from horizontal wires and cylinders to a surrounding fluid has received considerable interest in recent years. This reflects the wide range of application of such studies, including hot wire anemometry, electronic cooling and energy conservation. The investigations have dealt primarily with Newtonian fluids, particularly air and water. Relatively little attention has been paid to the free convection behavior of non-Newtonian fluids.

It is well known that most slurries and suspensions and some polymer solutions behave differently from Newtonian fluids. The most significant difference is the existence of a shear-rate-dependent viscosity. One class of purely viscous non-Newtonian fluids is the pseudoplastic fluid which exhibits a decreasing viscosity as the applied shear rate is increased. Recently, Shenoy and Mashelkar [1] reviewed the existing literature summarizing our knowledge of the free convection behavior of such non-Newtonian fluids. These fluids, which exhibit more complicated behavior than Newtonian fluids, are of importance in the chemical, pharmaceutical and food industries.

This paper reports the results of an experimental investigation of free convection from horizontal small diameter platinum wires to pseudoplastic aqueous polymer solutions [2].

THEORETICAL TREATMENT OF PSEUDOPLASTIC FLUIDS

Many mathematical models have been proposed to describe the rheological behavior of purely viscous aqueous polymer solutions. Among these the power law model is by far the simplest one. It relates the local shearing stress, τ, to the local shearing strain, $\dot{\gamma}$, by the simple relation

$$\tau = K(\dot{\gamma})^n \tag{1}$$

where K is the consistency index and n is the power law index. When $n = 1$, the relation reduces to the usual Newtonian relation. Although this relationship has the limitation that it does not describe the very high and very low shear rate range of pseudoplastic fluids it nevertheless has proven useful in describing the fluid behavior at intermediate shear rates normally encountered in engineering practice. In such cases the fluid mechanical and heat transfer behavior of purely viscous fluids under laminar flow conditions can often be predicted.

An example of this is given by the pioneering analytic work of Acrivos [3], who solved the problem of laminar free convection to power law fluids for a variety of simple geometries including the horizontal cylinder.

Restricting his analysis to the boundary layer regime Acrivos derived the following relationship for the average Nusselt number:

$$Nu = C\,Gr_K^{1/(2(n+1))}\,Pr_K^{n/(3n+1)} \tag{2}$$

where C is a constant which was evaluated for a vertical flat plate, a horizontal cylinder, a sphere and a vertical cone. Notice that when $n = 1$

$$Nu = C\,Gr^{1/4}\,Pr^{1/4} = C\,Ra^{1/4} \tag{3}$$

which may be deduced [4] from the laminar boundary layer equation for Newtonian fluids provided that $Pr \gg 1$.

An inspection of equation (2) reveals that the Nusselt number is a function of three independent variables, namely, the power law index, the Grashof number and the Prandtl number. Recently, Ng and Hartnett [5] transformed Acrivos' analytic results into a new set of dimensionless numbers yielding the following result:

$$Nu = C\,Ra_N^{1/(3n+1)}. \tag{4}$$

This simplifies the comparison of experimental and analytic studies since the Nusselt number is a function of only two independent variables, the Rayleigh number and the power law index provided that the Prandtl number is large. Furthermore, equation (4) brings out

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NOMENCLATURE

- A surface area [m^2]
- B temperature correction factor on shear rates, equation (5) [K]
- C coefficient in heat transfer correlation of power law fluids, equations (2)–(4)
- C_p heat capacity [$J\,kg^{-1}\,K^{-1}$]
- d characteristic length of the system, diameter of the wire [m]
- g gravitational acceleration [$m\,s^{-2}$]
- Gr Grashof number, $\rho^2 g\beta(T_w - T_f)d^3/\mu^2$
- Gr_K Grashof number introduced by Acrivos for power law fluids, $\rho^2 d^{n+2}[g\beta(T_w - T_f)]^{2-n}/K^2$
- h heat transfer coefficient [$W\,m^{-2}\,K^{-1}$]
- K consistency index, equation (1) [$N\,s^n\,m^{-2}$]
- k thermal conductivity of fluid [$W\,m^{-1}\,K^{-1}$]
- k_{pt} thermal conductivity of platinum wire [$W\,m^{-1}\,K^{-1}$]
- n power law index, equation (1)
- Nu Nusselt number, hd/k
- Pr Prandtl number, $\mu C_p/k$
- Pr_K Prandtl number introduced by Acrivos for power law fluids, $(\rho C_p/k)\,(K/\rho)^{2/(n+1)}d^{(1-n)/(1+n)} \times [g\beta(T_w - T_f)d]^{(3(n-1))/(2(n+1))}$
- Pr_N Prandtl number for power law fluids, $(K/k)C_p(\alpha/d^2)^{n-1}$
- q heat flux [W]
- Ra Rayleigh number, $\rho g\beta(T_w - T_f)d^3/\mu\alpha$
- Ra_N Rayleigh number for power law fluids $\rho g\beta(T_w - T_f)d^{2n+1}/K\alpha^n$
- T temperature [K]
- T_f fluid temperature [°C]
- T_m mean wire temperature [°C]
- T_w wall temperature [°C]
- ΔT temperature difference, $T_w - T_f$ [°C]
- wppm parts per million by weight.

Greek symbols

- α thermal diffusivity [$m^2\,s^{-1}$]
- β coefficient of volumetric expansion [K^{-1}]
- $\dot{\gamma}$ shear rate [s^{-1}]
- $\dot{\gamma}_r$ temperature reduced shear rate, equation (5) [s^{-1}]
- $\dot{\gamma}_w$ shear rate at the wall given by equation (8) [s^{-1}]
- μ Newtonian viscosity [Pa s]
- ρ density of the fluid [$kg\,m^{-3}$]
- τ shear stress [$N\,m^{-2}$].

more clearly the influence of the power law index n, on the heat transfer results. Using these newly derived dimensionless numbers, the overall experiments of Gentry and Wollersheim [6] were shown to be in good agreement with the prediction of Acrivos [3].

It should be noted that both the analytic study of Acrivos and the experimental study of Gentry and Wollersheim are in the region where the boundary layer thickness is small relative to the characteristic length. In contrast, the current experimental study of pseudoplastic fluids deals with free convection to horizontal wires, the diameters of which are of the same order or smaller than the free convection boundary layer.

EXPERIMENTAL DESIGN

Test fluids

Two commercial grades of the specially structured carboxypolymethylene or polyacrylic acid (Carbopol 960 and 934, B. F. Goodrich Company, Akron, Ohio) were used in the experimental study. Three different concentrations, 1000, 1500 and 1750 wppm of Carbopol 960 and three concentrations, 1000, 1250 and 1500 wppm of neutralized Carbopol 934 solutions were studied. Deionized water was used as the solvent.

Carbopol 960 is an ammonium salt, which is particularly effective as a thickening agent when added to water. It is an excellent choice for the study of purely viscous non-Newtonian fluids. In contrast Carbopol 934 is acidic and on addition to water the solution remains Newtonian up to relatively high concentrations of polymer. However, upon neutralization with a 10% sodium hydroxide solution (or 28% ammonium hydroxide) the resulting solution exhibits distinct pseudoplastic behavior. These aqueous Carbopol solutions give a power law index n ranging from 0.5 to 1.0 over a shear rate range from 10^{-2} to $10^0\ s^{-1}$.

Rheological property measurements

The steady shear rate viscosities of the polymer solutions as a function of shear rate were measured by a Weissenberg rheogoniometer (model R-18 from Sangamon Company, Sussex), a Brookfield viscometer (model LVT from Brookfield Company, Stoughton, Massachusetts) and a capillary viscometer. The Weissenberg rheogoniometer is capable of measuring the viscosity of the polymer solutions accurately at very low shear rate up to $10^{-2}\ s^{-1}$ for the test fluids used in the study. The main advantage of the Brookfield viscometer is that it is capable of measuring intermediate shear rate viscosities at different temperature levels. The capillary viscometer was used to measure the high shear rate viscosity at different temperature levels.

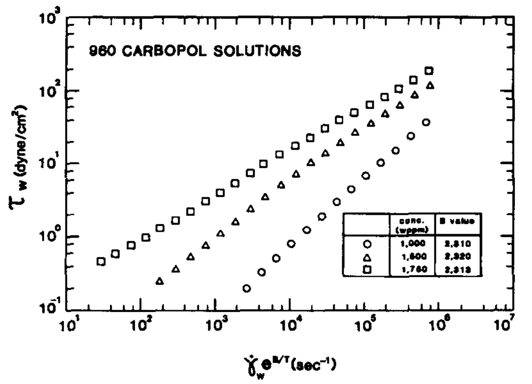

FIG. 1. Temperature-reduced flow data of Carbopol 960 solutions.

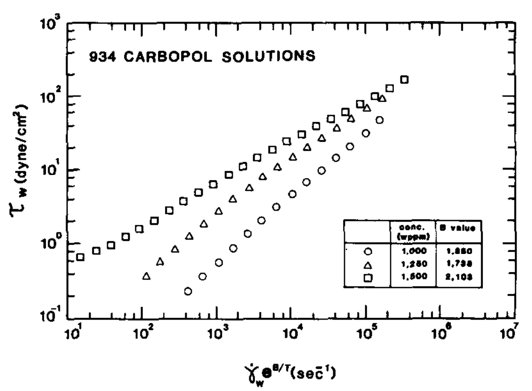

FIG. 2. Temperature-reduced flow data of neutralized Carbopol 934 solutions.

Using these three viscometers, the flow data of the polymer solutions at room temperature (23 ± 1°C), 40 and 55°C were measured. For each polymer concentration it was possible to reduce the flow curves for the three temperature levels to a single curve by introducing a so-called temperature reduced shear rate which was proposed by Christiansen *et al.* [7]

$$\dot{\gamma}_r = \dot{\gamma} \exp(B/T) \qquad (5)$$

where B was determined experimentally. The results are shown in Figs. 1 and 2 for the aqueous solutions of Carbopol 960 and 934. These flow curves can be used to determine the values of the consistency index, K, and the power law index, n, at any shear rate and temperature level for the various solutions studied.

Heat transfer apparatus

The heat transfer studies were carried out with three different diameter platinum wires of 0.0254, 0.0508 and 0.0826 cm, all 15.24 cm in length. These wires serve both as the principal heaters as well as the resistance thermometers to measure the surface temperature of the wire.

Each wire was placed, in turn, horizontally inside a rectangular stainless steel tank equipped with pyrex glass windows on two opposing sides. The tank, which is 10.2 cm wide, 30.5 cm long and 35.6 cm deep, is in turn placed in another stainless steel tank 30.5 × 45.7 × 40.8 cm also equipped with two glass windows on the same opposite sides. The four glass windows are aligned and a fluorescent lamp is located at the back of the outside tank so that the experiment can be viewed clearly through the windows.

The outside tank is filled with a clear transparent oil of high boiling point (Arcopak 70 from the Atlantic Richfield Oil Company, Los Angeles, California, boiling point 274°C) and serves as a constant temperature bath. Four 1000 W heating rods placed horizontally under the inner tank together with two 30.5 × 30.5 cm heating pads attached to the two window-less sides of the tank serve as the heat source for the constant temperature bath.

The heat source of the test wires is a series of direct current (d.c.) rechargeable batteries, which are capable of delivering a range of fixed voltages such that heat transfer under different values of heat flux can be studied. Block resistors are installed so that the resistance of the platinum wire can be measured accurately by passing very small currents through the wire. The current is accurately determined from the voltage drop measurement through a standard resistance of 0.1 Ω (Leeds and Northrup, North Wales, Pennsylvania) connected in series with the platinum wire. The current and the voltage drop across the test section, which are measured simultaneously by two digital multimeters (Hewlett-Packard Company, Palo Alto, California), are used to calculate both the heat flux and the resistance of the wire. Figure 3 shows a schematic diagram of the experimental set-up.

The platinum wires were calibrated in position in the test fluid tank by measuring the resistance of the test wires at specified temperature levels (measured by the calibrated thermocouples). Five temperature levels are covered in this way and a linear correlation is used to interpolate the temperature of the test section at intermediate temperature. It was recognized that the platinum wire may be easily contaminated by moisture or by a reducing atmosphere [8]. Such contamination may vary the temperature-dependent resistance of the platinum wire. Therefore, recalibration of the platinum wire was performed after each run with water and after each run with the polymer solutions.

The temperature of the fluid is measured by four copper–constantan thermocouples fitted in 0.3 cm thermowells which can be raised or lowered to cover a large area in the fluid. The tips of the thermowells are bent 90° to minimize conduction errors. In a typical heat transfer experiment these thermocouples are placed at locations 2–5 cm away from the test wires. The temperature readings of the test fluids from these four thermocouples agree with each other within ±0.2°C.

The electrical measurements determine the heat transfer from the test section to the fluid, as well as the mean platinum wire temperature. The surface temperature of the wire is then determined from the relation

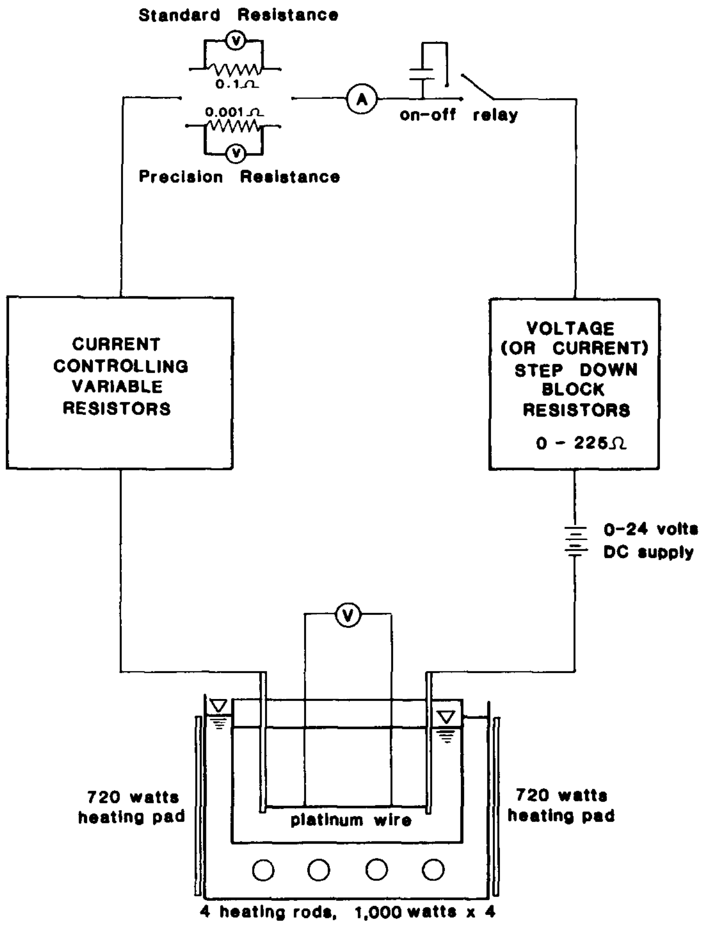

FIG. 3. Experimental set-up.

$$T_w = T_m - \frac{(q/A)d}{k_{pt}} \quad (6)$$

which assumes uniform heat generation in the wire and constant thermal conductivity of the platinum.

Finally the heat transfer coefficient can be determined from the expression

$$h = \frac{\cdot\, q/A}{T_w - T_f}. \quad (7)$$

Calibration free convection runs with water as the test fluid were carried out for each of the three platinum wires. A total of 579 runs were made covering a range of Rayleigh numbers from 1 to 750, while the Prandtl number varied from 3 to 7. These results, shown in Fig. 4 in the form of Nusselt number vs Rayleigh number, are within 7% of the correlation proposed by Morgan [9].

Having established the reliability of the experimental procedure, free convection heat transfer from the three diameter platinum wires to the pseudoplastic aqueous polymer solutions were then carried out.

HEAT TRANSFER RESULTS

The experimental values of the heat transfer coefficient for the polymer solutions were determined in a straightforward manner, as in the case of water. Inspection of the governing equations reveals that the dimensionless heat transfer coefficient expressed as the Nusselt number, should be a function of the Rayleigh number Ra_N, and the power law index n if the Prandtl number Pr_N of the fluid is sufficiently high [2]. This condition of high Prandtl number is generally satisfied by the pseudoplastics being investigated.

These dimensionless numbers contain such physical properties of the fluid as density ρ, volumetric expansion coefficient β, heat capacity C_p and thermal conductivity k. These properties, which were evaluated at the mean film temperature, were taken to be the same as those of water [10–12]. In the case of the rheological properties K and n which are also needed to determine Ra_N it is necessary to specify the temperature level and the wall shear rate. Fortunately the value of the Rayleigh number, Ra_N, is not too sensitive to the wall shear rate. This is seen on Fig. 5 which presents the quantity $Ra_N/\rho g\beta\Delta T$ as a function of $\dot{\gamma}_w$ at three temperature levels and for the three diameter wires for the 1500 wppm Carbopol 960 solution. Similar results were found for the other solutions studied.

In view of the relative insensitivity of the Rayleigh number, Ra_N, to the wall shear rate it was decided to use the prediction of Acrivos as the estimate of the

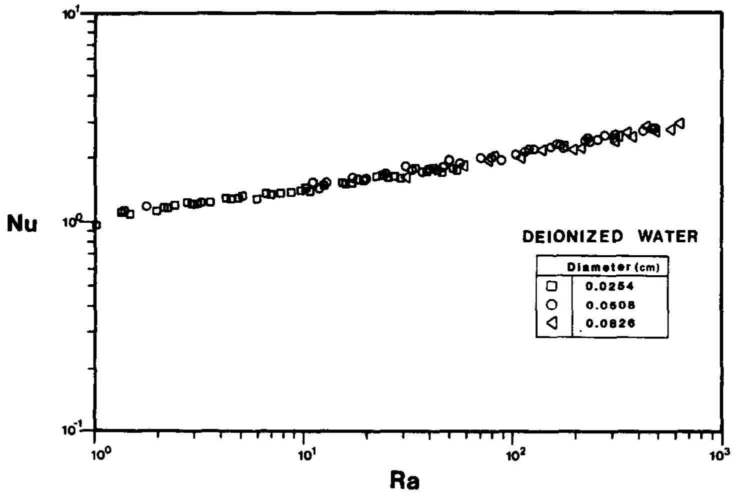

FIG. 4. Calibration free convection results—horizontal wires to water.

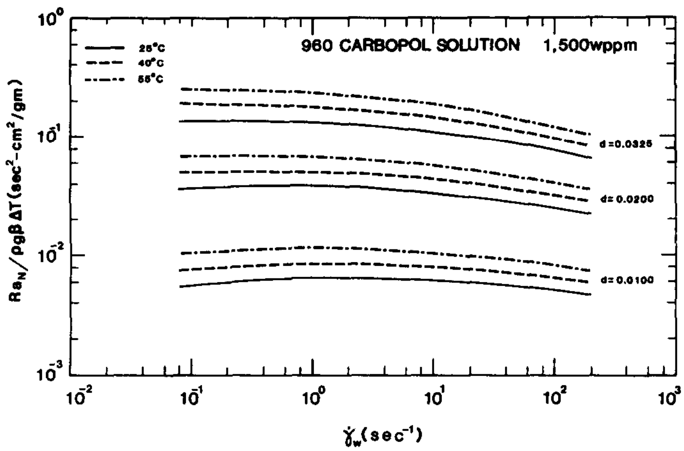

FIG. 5. Evaluation of Ra_N as a function of wall shear rate.

wall shear rate for the evaluation of the rheological properties, n and K

$$\dot{\gamma}_w \sim \left[\frac{g\beta(T_w - T_f)}{d}\right]^{1/2} [Ra_N^{(5-3n)/2(3n+1)}/Pr_N^{1/2}]. \quad (8)$$

The resulting Nusselt numbers for the three platinum wires to the three concentrations of Carbopol 960 and three concentrations of neutralized Carbopol 934 are given on Figs. 6 and 7. In the range of $Ra_N = 10^{-3}$–10^0 the Nusselt number is seen to decrease with decreasing values of the power law index n at a fixed value of Ra_N. This contrasts with the experimental results of Gentry and Wollersheim [6] obtained at high Rayleigh numbers. However, it is consistent with the general form of Acrivos' boundary layer prediction ($Nu\ Ra_N^{1/(3n+1)}$) increasing with decreasing values of n at $Ra_N > 1$ but decreasing with decreasing values of n at $Ra_N < 1$. Since the current results lie outside the region where the boundary layer assumptions are valid it is not surprising that they are not in agreement with Acrivos' prediction.

A statistical analysis of the experimental data was carried out and resulted in the following best-fit curve:

$$Nu = (0.761 + 0.413n)Ra_N^{0.5/(3n+1)}. \quad (9)$$

This is shown in Fig. 8. Of the 301 experimental measurements 239 are within 10% of equation (9) and 294 fall within 20% of the proposed correlation. In the special case of $n = 1$ for Newtonian fluids the correlation becomes

$$Nu = 1.192Ra^{0.125} \quad (10)$$

which is consistent with the high Prandtl number predictions of Fand and Brucker [13] and the measure-

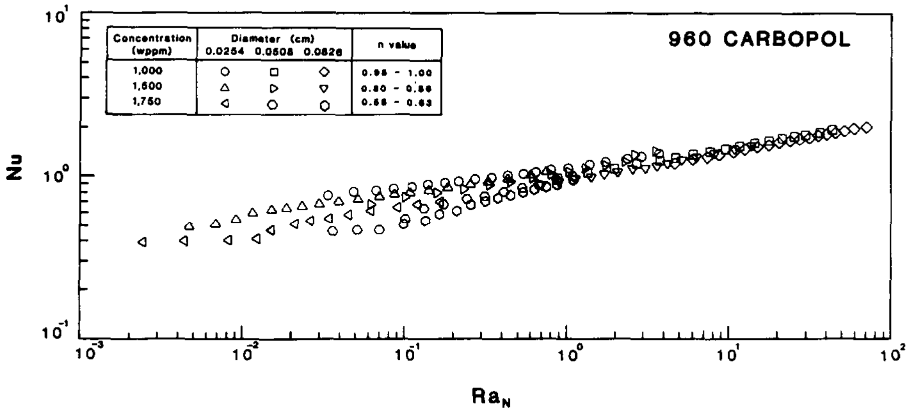

FIG. 6. Free convection heat transfer data of Carbopol 960 solutions.

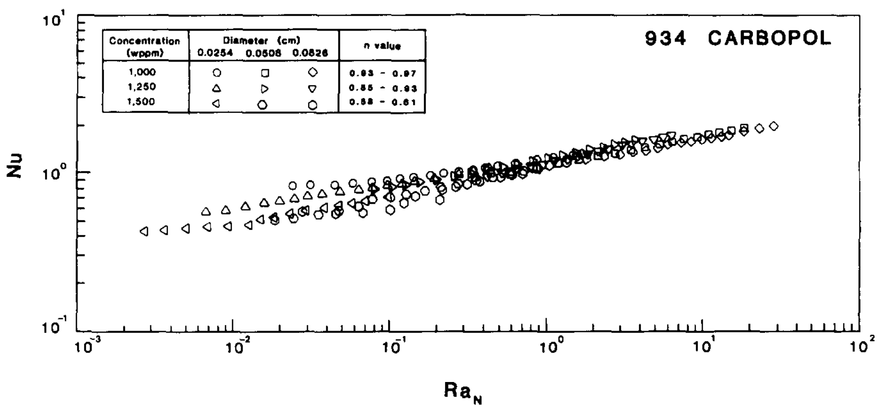

FIG. 7. Free convection heat transfer data of neutralized Carbopol 934 solutions.

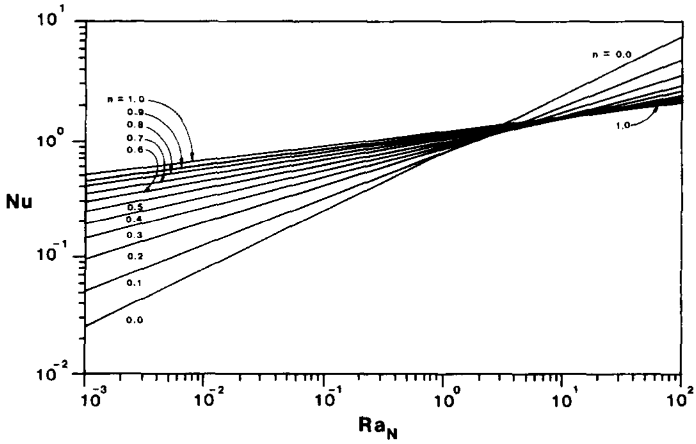

FIG. 8. Proposed correlation for free convection heat transfer from horizontal wires to pseudoplastic fluids.

ments of Ng *et al.* for high Prandtl number viscoelastic fluids [14].

CONCLUSIONS

The following findings can be drawn from this study.

(1) In the case of free convection heat transfer from horizontal cylinders to pseudoplastic fluids the Nusselt number can be correlated as a function of Ra_N and n.

(2) In the range of Rayleigh numbers smaller than unity, the dimensionless free convection heat transfer for a horizontal wire to a power law fluid decreases as the power law index decreases at a fixed Rayleigh number. This agrees with the functional form of Acrivos' analytic solution.

(3) Based on the experimental data which covered the range of Ra_N from 10^{-2} to 10^2 and power law index n from 0.5 to 1 the following correlation equation is proposed

$$Nu = (0.761 + 0.431n)Ra_N^{0.5/(3n+1)}.$$

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TRANSFERT THERMIQUE PAR CONVECTION NATURELLE ENTRE DES FILS HORIZONTAUX ET DES FLUIDES PSEUDOPLASTIQUES

Résumé—Des valeurs expérimentales de nombre de Nusselt sont données pour le transfert de chaleur par convection naturelle entre des fils horizontaux de petit diamètre et des fluides pseudoplastiques.

WÄRMEÜBERGANG BEI FREIER KONVEKTION VON HORIZONTALEN DRÄHTEN AN PSEUDOPLASTISCHE FLUIDE

Zusammenfassung—Es werden experimentell ermittelte Nusselt-Zahlen für den Wärmeübergang bei freier Konvektion von dünnen horizontalen Drähten an pseudoplastische Fluide mitgeteilt.

СВОБОДНОКОНВЕКТИВНЫЙ ТЕПЛОПЕРЕНОС ОТ ГОРИЗОНТАЛЬНЫХ ПРОВОЛОЧЕК К ПСЕВДОПЛАСТИЧНЫМ ЖИДКОСТЯМ

Аннотация—Представлены экспериментальные значения числа Нуссельта для свободноконвективного теплопереноса от горизонтальных проволочек к псевдопластиным жидкостям.